POLYPHASE QUADRATURE FILTERS – A NEW SUBBAND CODING TECHNIQUE

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ABSTRACT

A new method of implementing filter banks for subband coding of speech is presented. First, Esteban's Quadrature Mirror Filter principle is extended to allow the direct synthesis of filter banks with any number of equal size filter bands. Then, by combining the quadrature filter characteristic with the polyphase network implementation of filter banks, a new filter bank structure is obtained which requires 35% fewer computations than existing designs.

In this paper the theory of operation of the Polyphase Quadrature Filter is presented and techniques for its efficient implementation are described. Then, examples of filter banks using this approach are shown and compared with other designs, and a simulation of a 16 Kbits/s coder using these filters is presented.

INTRODUCTION

The technique of subband coding has been shown to be capable of near toll quality speech transmission at 16 Kbits/s. A disadvantage of SBC relative to other coding techniques, however, is the high amount of computation required in a digital implementation of the subband splitting and reconstruction filter banks.

One approach to subband coding of speech signals uses a filter bank of 8 equal size bands followed by adaptive differential quantization or block quantization of the desampled bandpass filter outputs.[1,2] These filter banks may be implemented using a three stage cascade of Quadrature Mirror Filter pairs[3] to successively divide the input signal spectrum in half and desample by 2. In practice, approximation procedures are used to design parallel filter banks which approximate the performance of the QMF cascade. These approximations reduce the computation rate by reducing the filter length and also by sharing computations between pairs of filters. Disadvantages of this approach are that the computation rate is still high (160 000 multiplies per second in one case[1]), optimization must be performed on each bandpass filter as well as the original QMF pair, and the filter bank size is restricted to powers of two.

This paper presents a more efficient structure for the implementation of banks of equal size bandsplitting and reconstruction filters. This structure, the multiband quadrature filter, is a variation of the FFT based filter bank which was originally proposed by Darlington[4] and later refined by Bellanger[5] and others[6] in the polyphase filter implementation of telephone transmultiplexers.

THEORY OF OPERATION

Since its development by Esteban[3], the QMF pair has been used as the basis for most SBC filter banks because it has the desirable property that a signal may be divided into two equal bands and desampled by two without loss of information. That is, the two filter outputs may be recombined using a complementary filter pair with arbitrarily small distortion of the signal. This property is a result of two filter characteristics:

(1) The aliasing components introduced by the desampling process cancel when the two bands are recombined, and

(2) The filter frequency responses overlap and add in such a way that the composite frequency response is nearly flat for all frequencies.

For a multiband filter, the corresponding requirements are that cancellation of aliasing occur between adjacent bands and that the filter shapes be controlled such that the transition bands of adjacent filters add to produce a flat response.

The following derivation demonstrates that a filter bank with these characteristics can be formed by first designing a lowpass prototype filter with a controlled transition band frequency response. The filter bank is conceptually formed by multiplying the impulse response of this lowpass prototype by sinusoids at frequencies corresponding to the center frequencies of the desired filters. It will be shown that proper phasing of the different sinusoids allows cancellation of aliasing to be achieved, while proper design of the lowpass prototype produces nearly flat frequency response at signal reconstruction.

In the following analysis, a sampling rate of 1 is assumed, so the frequency range to be covered is 0 to π radians/sec. A bank of M filters is to be synthesized, as shown in Fig. 1. The nominal bandwidth of each filter is π/M, and the filter center frequencies are at odd multiples of $\pi/2M$.

Each bandpass filter is conceptually formed by translating a lowpass filter H(z) of bandwidth $\pi/2M$ to the desired center frequency, forming the bandpass filter $H_i(z)$ or $K_i(z)$ in

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Fig. 1. For analytical purposes, this real bandpass filter will be viewed as the sum of two complex filters $F_i(z)$ and $G_i(z)$ located respectively at the positive and negative center frequencies.

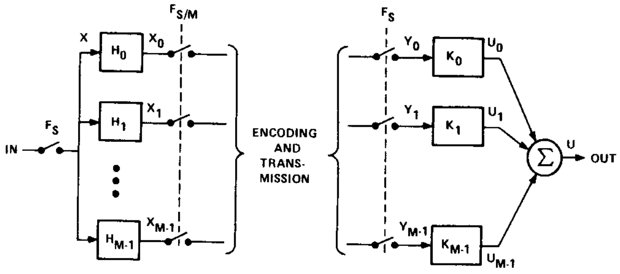

FIGURE 1. Desampling and reconstruction filter banks for Subband Coding.

If the prototype filter is a FIR filter with impulse response h(n) and z transform H(z), the complex bandpass filters are

$$F_i(z) = H(e^{-j\pi(2i+1)/2M}z) \quad (1a)$$

$$G_i(z) = H(e^{+j\pi(2i+1)/2M}z) \quad (1b)$$

and the composite filters are

$$H_i(z) = a_iF_i(z) + b_iG_i(z) \quad (2a)$$

$$K_i(z) = c_iF_i(z) + d_iG_i(z) \quad (2b)$$

where a_i, b_i, c_i, and d_i are complex constants. If $a_i = b_i^*$, the impulse response of H_i is real and given by

$$h_i(n) = \{Re(a_i)\cos[\pi(2i+1)(2n+1)/4M] - Im(a_i)\sin[\pi(2i+1)(2n+1)/4M]\}\, h(n) \quad (3)$$

Replacing a_i and b_i with c_i and d_i, a corresponding equation may be obtained for $k_i(n)$.

Referring to Fig. 1, the input signal is X(z) and the output from filter H_i is

$$X_i(z) = H_i(z)X(z) \quad (4)$$

Desampling by M before transmission and subsequent interpolation by M produces the signal $Y_i(z)$ which has the z transform[7]

$$Y_i(z) = M^{-i}\sum_{l=0}^{M-1}[a_iF_i(e^{-j2\pi l/M}z) + b_iG_i(e^{-j2\pi l/M}z)]\,.\; X(e^{-j2\pi l/M}z) \quad (5)$$

This equation shows that the spectrum of Y_i consists of the filtered input signal (corresponding to l=0) along with images of the signal spaced $2\pi/M$ apart, as shown in Fig. 2.

It is assumed that the filter K_i is sufficiently sharp that all components of Y_i will be eliminated except the original signal band and the edges of the images adjacent to the higher and lower passband edges, as shown in Fig. 2d. It can easily be shown

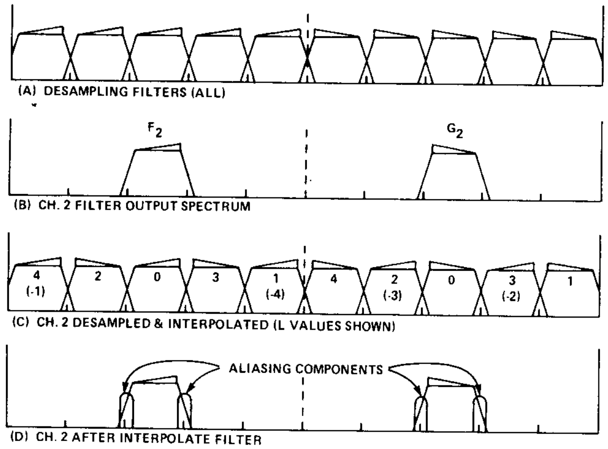

FIGURE 2. Frequency domain view of the operation of 5 band desampling and interpolation filters.

that images of the output of complex filter F_i corresponding to values of $l = -i$ and $l = -(i+1)$ lie at the low and high edges, respectively, of the signal band. Similarly, images of the output of G_i at $l=i$ and $l=i+1$ are adjacent to the desired band. Therefore, after eliminating those terms of Eq. 5 which are completely eliminated by filter K_i, the signal at the filter output is

$$\begin{aligned} U_i(z) = K_i(z)Y_i(z) = M^{-1}K_i(z)\{&a_iF_i(z)X(z) + b_iG_i(z)X(z) \\ &+ a_iF_i(e^{j2\pi i/M}z)X(e^{j2\pi i/M}z) \\ &+ b_iG_i(e^{-j2\pi i/M}z)X(e^{-j2\pi i/M}z) \\ &+ a_iF_i(e^{j2\pi(i+1)/M}z)X(e^{j2\pi(i+1)/M}z) \\ &+ b_iG_i(e^{-j2\pi(i+1)/M}z)X(e^{-j2\pi(i+1)/M}z)\} \end{aligned} \quad (6)$$

where the first two terms represent the original signal, the third and fourth terms represent aliasing at the low edge, and the last two terms represent aliasing at the high edge. Note that Eq. 6 is valid for i from 1 to M-2. For band 0 the low edge aliasing components are absent while for band M-1, the high edge aliasing components are absent.

We wish to synthesize the filters in such a way that the aliasing components at the low edge of one band exactly cancel the components at the high edge of the preceding band. This condition is achieved if

$$\begin{aligned} &K_i(z)\{a_iF_i(e^{j2\pi i/M}z)X(e^{j2\pi i/M}z) + b_iG_i(e^{-j2\pi i/M}z)X(e^{-j2\pi i/M}z)\} \\ &= -K_{i-1}(z)\{a_{i-1}F_{i-1}(e^{j2\pi i/M}z)X(e^{j2\pi i/M}z) \\ &\quad + b_{i-1}G_{i-1}(e^{-j2\pi i/M}z)X(e^{-j2\pi i/M}z)\} \end{aligned} \quad (7)$$

which is satisfied if

$$a_iK_i(z)F_i(e^{j2\pi i/M}z) = -a_{i-1}K_{i-1}(z)F_{i-1}(e^{j2\pi i/M}z) \quad (8a)$$

$$b_iK_i(z)G_i(e^{-j2\pi i/M}z) = -b_{i-1}K_{i-1}(z)G_{i-1}(e^{-j2\pi i/M}z) \quad (8b)$$

By substituting Equations 1 and 2b into 8a and simplifying, this equation reduces to

$$a_i c_i H(e^{-j\pi(2i+1)/2M}z) H(e^{j\pi(2i-1)/2M}z) + a_i d_i H(e^{j\pi(2i+1)/2M}z) H(e^{j\pi(2i-1)/2M}z)$$

$$= -a_{i-1} c_{i-1} H(e^{-j\pi(2i-1)/2M}z) H(e^{j\pi(2i+1)/2M}z) - a_{i-1} d_{i-1} H(e^{j\pi(2i-1)/2M}z) H(e^{j\pi(2i+1)/2M}z) \quad (9)$$

The expressions

$H(e^{-j\pi(2i+1)/2M}z)$ and

$H(e^{j\pi(2i-1)/2M}z)$

may be viewed as images of the lowpass prototype appearing respectively below and above the Nyquist frequency. If i is greater than zero, these images do not overlap in frequency, so their product is zero and the first term on the left side of Eq. 9 may be eliminated. By a similar argument, the first term on the right side is zero also.

The remaining terms in Eq. 9 can be seen to be equal if $a_i d_i = -a_{i-1} d_{i-1}$, or equivalently,

$$a_i c_i^* = -a_{i-1} c_{i-1}^* \quad (10)$$

A similar derivation will show that the relationship of Eq. 10 also satisfies Eq. 8b.

Assuming perfect cancellation of aliasing, the output signal is

$$U(z) = \sum_{1=0}^{M-1} [a_i F_i(z) + a_i^* G_i(z)] [c_i F_i(z) + c_i^* G_i(z)] X(z) \quad (11)$$

For i from 1 to M-2 the product of F_i and G_i is zero, so (11) may be rewritten

$$U(z) = (a_0 c_0^* + a_0^* c_0) F_0(z) G_0(z) + (a_{M-1} c_{M-1}^* + a_{M-1}^* c_{M-1}) F_{M-1}(z) G_{M-1}(z) + \sum_{i=0}^{M-1} [a_i c_i F_i^2(z) + a_i^* c_i^* G_i^2(z)] X(z) \quad (12)$$

To maintain flat phase response for the composite system, it is desired that $a_i c_i = 1$, or assuming both have unit magnitude,

$$a_i = c_i^* \quad (13)$$

A final constraint on the weights is that the first two terms of Eq. 12 must disappear in order to achieve flat response at 0 and $F_s/2$. The corresponding constraint is

$$a_0^4 = -1 \quad \text{and} \quad a_{M-1}^4 = -1 \quad (14)$$

where unit magnitude is again assumed.

In summary, Eq. 14 defines one of 4 possible values to be inserted in Eq. 3 to synthesize the band 0 bandsplitting filter.

Combining Equations 10 and 13, the higher order filters may be determined by the recursive relationship

$$a_i = \pm j a_{i-1} \quad (15)$$

Finally, the reconstruction filters are determined from the corresponding bandsplitting filters using the relationship of Eq. 13.

IMPLEMENTATION

One efficient implementation of a multiband quadrature filter bank is shown in Fig. 3. As indicated, the stored input values are read from memory and multiplied by the coefficients of the lowpass prototype filter h(n). Since the sinusoid in Eq. 3 contains an odd number of half cycles in 2M points, blocks of 2M products of the multiplications are accumulated with the sign of alternate blocks negated. These 2M values are then multiplied by M sinusoids to generate the M output values.

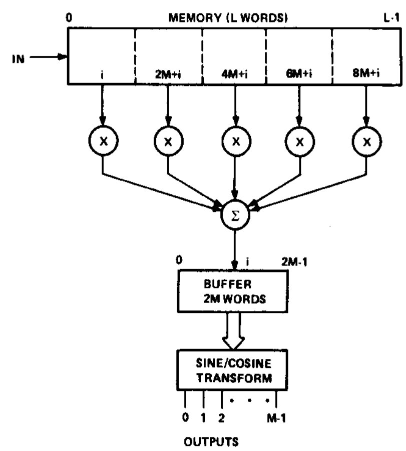

FIGURE 3. Structure for efficient implementation of a multiband quadrature filter bank.

An even more efficient implementation is possible when this technique is used to process voice signals. Since flat response to DC is not required, the constraint of Eq. 14 is unnecessary. With this constraint removed, the coefficient a_i can be either pure real or pure imaginary, so Eq. 3 will contain either the sine or cosine term but not both. A further simplification of the implementation is then possible. By taking advantage of the symmetry and antisymmetry of the cosine and sine, respectively, the number of multiplications in a DFT implementation is reduced to M^2 rather than $2M^2$.

EXAMPLES

Two filter banks were synthesized and simulated. The first is an 8 band filter bank based on a 40 tap lowpass prototype filter. This design allows a direct comparison of performance and computation with a design derived from QMF pairs. The second design is a 5 band design using a 30 tap prototype.

In both cases the lowpass prototype was designed using the Hooke and Jeaves optimization algorithm, which was originally applied to the design of QMF pairs by Johnston[8].

The filters are designed to satisfy as closely as possible the relations

$$H^2(\omega) = 1 - H^2(\pi/M - \omega) \quad , 0 \leqslant \omega \leqslant \pi/M \qquad (16a)$$

$$\text{and} \quad H^2(\omega) = 0 \quad , \pi/M < \omega \leqslant \pi \qquad (16b)$$

The resulting lowpass prototype frequency responses are shown in Fig. 4.

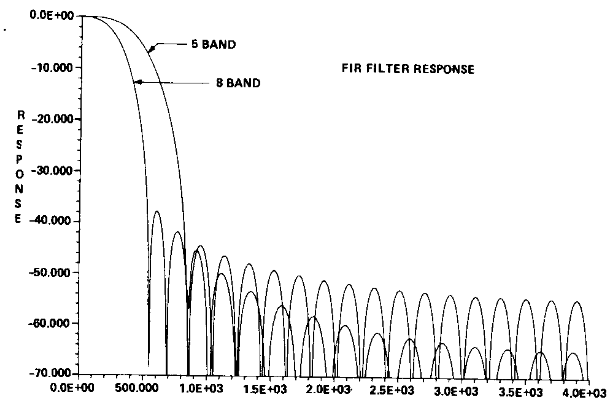

FIGURE 4. Simplified "DFT" structure for use when flat response at DC is not required.

The 8 band filter bank was tested in a completely flat version and a version with zero response at the endpoints as previously discussed. The flat version uses the definitions

$$a_i = \begin{cases} .707 + j.707 & , i = 0, 2, 4, \ldots \\ .707 - j.707 & , i = 1, 3, 5, \ldots \end{cases} \qquad (17)$$

While the other uses

$$a_i = \begin{cases} j & , i = 0, 2, 4, \ldots \\ 1 & , i = 1, 3, 5, \ldots \end{cases} \qquad (18)$$

The resulting frequency response for bandsplitting and reconstruction is shown in Fig. 5 for both versions of the 8 band filter.

A complete subband coder simulation was developed using the 8 band nonflat filter bank. An 8 kHz sampling rate was used with block quantization[1,2] to achieve a 16 Kbit/s data rate. Initial listening tests have confirmed that minimal degradation of the speech signal is produced by this technique.

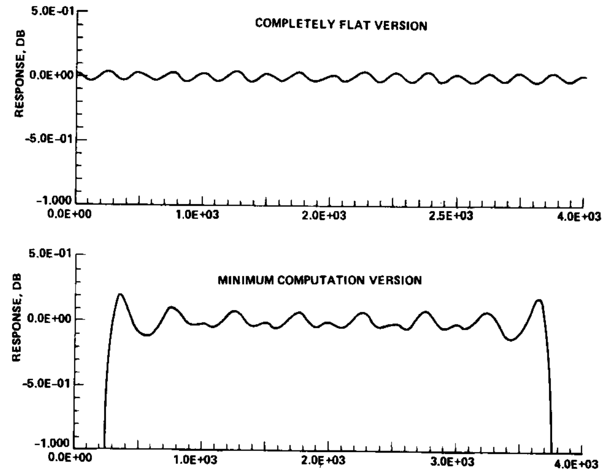

FIGURE 5. Frequency response of 30 tap 5 band and 40 tap 8 band lowpass prototypes.

The 8 band bandsplitting filter bank requires 40 multiplications per set of output points to perform windowing and 64 multiplications for the DFT using a brute force approach. The total multiplication rate is then 104 000 multiplies/s, which is a 35% improvement over an equivalent half band structure. The 5 band bank requires only 88 000 mpy/s, but testing is not complete.

SUMMARY

A new filter bank structure for use in the subband coding of speech has been described. Its primary advantage is a reduction in the amount of computation required to implement the filter bank. A secondary advantage is that a filter bank with any number of equal size filters can be produced using this approach.

One area for further study is to determine the relative benefits of different numbers of filter bands. As noted, some computation savings can be achieved by reducing the number of bands from eight to five, but the dependence of voice quality on the number of bands has not yet been determined.

REFERENCES

[1] D. Esteban and C. Galand, "Multiport Implementation of Real Time 16 KBPS Sub-Band Coder," *IEEE Int. Conf. on Acoust., Speech, Signal Proc.*, pp. 459-462, 1981.

[2] R. Cheung and R. Winslow, "High Quality 16 KB/S Voice Transmission: The Subband Coder Approach," *IEEE Int. Conf. on Acoust., Speech, Signal Proc.*, pp. 319-322, 1980.

[3] D. Esteban and C. Galand, "Application of Quadrature Mirror Filters to Split Band Voice Coding Schemes," *IEEE Int. Conf. on Acoust., Speech, Signal Proc.*, pp. 191-199, 1977.

[4] S. Darlington, "On Digital Single-Sideband Modulators," *IEEE Trans. Circuit Theory*, Vol. CT-17, pp. 409-414, Aug. 1970.

[5] G. Bonnerot, M. Coudreuse, and M.G. Bellanger, "Digital Processing Techniques in the 60 Channel Transmultiplexer," *IEEE Trans. Communications,"* Vol. COM-26, No. 5, pp. 698-706, May 1978.

[6] C.M. Puckette and T.G. Marshall, Jr., Editors, Special Issue on Transmultiplexers of *IEEE Trans. Communications,* Vol. COM-30, No. 7, July 1982.

[7] R. Schaffer and L. Rabiner, "A Digital Signal Processing Approach to Interpolation," *Proc. IEEE,* Vol. 61, pp. 692-702, June 1973.

[8] J. Johnston, "A Filter Family Designed for Use in Quadrature Mirror Filter Banks," *IEEE Int. Conf. on Acoust., Speech, Signal Proc.*, pp. 291-294, 1980.